

# The Crystal Structure of Potassium Metavanadate Monohydrate, $\text{KVO}_3 \cdot \text{H}_2\text{O}$ \*

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$\text{KVO}_3 \cdot \text{H}_2\text{O}$  is orthorhombic,  $Pnam$ ,  $a = 8.15_1$ ,  $b = 13.58_8$ ,  $c = 3.69$ , Å,  $Z = 4$ . A trial structure, established by the use of a vector-shift method applied to the Patterson projection on (001), was refined by electron-density projections (including bounded projections) and by least-squares analysis. In the structure each vanadium atom is linked to five oxygen atoms to form a distorted trigonal dipyramid; the polyhedra so formed share edges to form continuous chains parallel to the  $c$  axis. This fivefold coordination is analogous to that which exists in  $\text{V}_2\text{O}_5$ .

## Introduction

When vanadium pentoxide is dissolved in a solution of potassium hydroxide, and the solution is maintained at a pH between 6.5 and 8, clear colorless needles of both potassium metavanadate,  $\text{KVO}_3$ , and its monohydrate,  $\text{KVO}_3 \cdot \text{H}_2\text{O}$ , are readily produced on concentration and cooling. This pH range comprises the so-called 'metavanadate' stability range, in contrast with the range of pH > 10 corresponding to the 'orthovanadates', the range from pH 8 to 10 corresponding to the 'pyrovanadates', and the range corresponding to the orange 'polyvanadates' from pH 6.5 to the isoelectric point at pH 1.6, at which point brown  $\text{V}_2\text{O}_5$  hydrates precipitate. The system  $\text{Na}_2\text{O}-\text{V}_2\text{O}_5-\text{H}_2\text{O}$ , which is characterized in a general way by the stability ranges referred to, has been the subject of considerable study of various workers and by various physical chemical methods [see for example Düllberg (1903), Jander & Jahr (1933), Souchay & Carpeni (1946), Ducret (1951)]. Although the reactions involved are usually considered to be a series of successive condensations toward higher molecular weight complexes with increasing acidity, no details have been established to date concerning any of the molecular structures, or any of the mechanisms involved. In the U.S. Geological Survey laboratories we are making an attempt to approach the problem of the constitution and interrelation of the many phases present in the system  $\text{K}_2\text{O}-\text{V}_2\text{O}_5-\text{H}_2\text{O}$  by means of crystal-structure analysis of the solids that appear. In this paper the crystal structure of  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  is described in detail [a preliminary account has been given in Christ, Clark & Evans (1953)]. A study of the structure of  $\text{KVO}_3$  has been completed and will be described in a forthcoming article.

$\text{KVO}_3$  and  $\text{KVO}_3 \cdot \text{H}_2\text{O}$ , although their crystal structures have been revealed to be entirely different, are similar in chemical and physical properties and mode of genesis. Both are sparingly soluble in cold water and

readily soluble in hot water, and both have pronounced fibrous cleavage. When a potassium metavanadate solution is rapidly cooled, a crystalline precipitate characterized by fine hairlike needles appears, which is mainly  $\text{KVO}_3$ . As the rate of cooling slows at lower temperatures, needles of similar habit of  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  are also formed. On very slow crystallization by evaporation, radiating groups of blunt rods of  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  are produced, sometimes simultaneously and in contact with stubby, pseudo-octahedral crystals of  $\text{KVO}_3$ .  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  apparently converts to  $\text{KVO}_3$  on grinding.

$\text{KVO}_3 \cdot \text{H}_2\text{O}$  was first recognized by Norblad (1875) and was noted by Fock (1889), but otherwise to our knowledge it is not mentioned in the literature.

## Experimental work

### *Preparation of crystals and chemical analysis*

At the beginning of this investigation it was believed that the two compounds crystallizing in the pH range between 6.5 and 8 were polymorphic forms of  $\text{KVO}_3$ . The structure analysis, however, soon showed clearly that the substance dealt with here was a monohydrate and was entirely consistent with a compound of formula  $\text{KVO}_3 \cdot \text{H}_2\text{O}$ . With this in mind it was then possible to resolve the difficulties in chemical analysis that arose from dealing with a mixture. By very slow crystallization, mixtures were prepared containing crystals sufficiently large to ensure efficient separation. An analysis of the  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  obtained in this way is given below:

	Found (%)	Theoretical (%)
$\text{K}_2\text{O}$	30.24	30.18
$\text{V}_2\text{O}_5$	58.40	58.28
$\text{H}_2\text{O}$	11.64	11.54
Total	100.28	100.00

(Analyst, George B. Magin, Jr., U.S. Geological Survey.)

The typical blunt-rod habit of  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  is shown in Fig. 1.

\* Publication authorized by the Director, U.S. Geological Survey.

*Space group and unit-cell dimensions*

Zero- and upper-level photographs around [001], made on both Weissenberg and precession cameras and

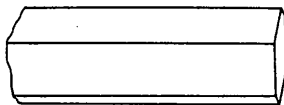


Fig. 1. Typical blunt-rod crystal of  $\text{KVO}_3 \cdot \text{H}_2\text{O}$ .

with both zirconium-filtered  $\text{Mo } K\alpha$  ( $\text{Mo}/\text{Zr}$ ) and nickel-filtered  $\text{Cu } K\alpha$  ( $\text{Cu}/\text{Ni}$ ) radiations, were used to establish the lattice type and symmetry. Systematic extinctions were found to be of the type  $h0l$ ,  $h \neq 2n$ , and  $0kl$ ,  $k+l \neq 2n$ . These lead to the space groups  $Pnam-D_{2h}^{16}$  or  $Pna-C_{2v}^9$ . Visual examination of the reflections obtained on rotation patterns made around [001] establishes that corresponding reflections on all even-layer lines are similar, as are those on all odd-layer lines. It follows that in this structure most atoms

Table 1. *X-ray powder data for  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  and  $\text{KVO}_3$*

These data correspond to a mixture of  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  and  $\text{KVO}_3$  that is obtained when crystals of  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  are powdered. The  $d_{hkl}$  values for  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  are calculated from the lattice constants given in the text; the  $d_{hkl}$  values for  $\text{KVO}_3$  are derived from the following data: orthorhombic  $a = 5.70$ ,  $b = 10.82$ ,  $c = 5.22$  Å. The lines were indexed with the help of a  $\text{KVO}_3$  powder pattern.  $\text{Cu}/\text{Ni}$  radiation  $\lambda = 1.5418$  Å was used. Data are listed only for  $d_{hkl} > 2.00$  Å. The lines corresponding to  $\text{KVO}_3$  are so indicated, the other lines being due to  $\text{KVO}_3 \cdot \text{H}_2\text{O}$ .

Measured		Calculated		
<i>I</i>	$d_{hkl}$ (Å)	$d_{hkl}$ (Å)	<i>hkl</i>	
15	7.00	6.99	110	
5	5.39	5.41	020	$\text{KVO}_3$
9	5.20	5.22	001	$\text{KVO}_3$
		5.22	120	
10	3.910	3.904	210	
		3.924	120	$\text{KVO}_3$
8	3.748	3.756	021	$\text{KVO}_3$
10	3.488	3.495	220	
7	3.262	3.268	111	
14	3.128	3.135	140	
		3.136	121	$\text{KVO}_3$
11	3.024	3.017	121	
		3.030	230	
13	2.854	2.850	200	$\text{KVO}_3$
		2.864	031	
4	2.735	2.738	201	
11	2.680	2.684	211	
		2.702	131	
8	2.607	2.609	240	
		2.610	002	$\text{KVO}_3$
7	2.526	2.521	220	$\text{KVO}_3$
		2.523	320	
8	2.440	2.437	211	$\text{KVO}_3$
5	2.402	2.402	041	$\text{KVO}_3$
7	2.342	2.343	231	
7	2.302	2.318	112	$\text{KVO}_3$
9	2.257	2.261	250	
		2.264	060	
4	2.179	2.182	160	
4	2.125	2.122	340	
3	2.043	2.038	400	

are situated on planes parallel to (001) and  $\frac{1}{2}c$  apart. This fact suggests that all the atoms lie on mirror planes in the space group  $Pnam$ , and on this assumption a satisfactory structure has been determined.

Lattice-constant measurements were made with a precession camera, the crystal-to-film distance of which had been accurately calibrated by use of a quartz crystal. Patterns of the ( $h0l$ ) and ( $0kl$ ) zones were prepared using  $\text{Mo}/\text{Zr}$  radiation and were corrected for horizontal and vertical shrinkage. The values of the cell edges derived from these patterns were checked against those derived from a powder pattern made with  $\text{Cu}/\text{Ni}$  radiation. The powder data are given in Table 1. As  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  converts to  $\text{KVO}_3$  on grinding, it was impossible to prepare a powder pattern of  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  only; the data of Table 1 correspond to the mixture. The crystallographic data for  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  are given below:

Orthorhombic; space group:  $Pnam-D_{2h}^{16}$ .

$a = 8.15_1 \pm 0.008$ ,  $b = 13.58_3 \pm 0.010$ ,  $c = 3.69_7 \pm 0.004$  Å.

( $\text{Mo } \lambda$ :  $K\alpha = 0.71069$  Å;  $K\alpha_1 = 0.70926$  Å).

Cell contents:  $4(\text{KVO}_3 \cdot \text{H}_2\text{O})$ .

Density (calc.) =  $2.53$  g.cm.<sup>-3</sup>, density (obs.) =  $2.52$  g.cm.<sup>-3</sup>.

*Intensity measurements*

For the intensity measurements multiple-film Weissenberg patterns using  $\text{Mo}/\text{Zr}$  radiation were prepared. Three films interleaved with 0.0005 in. Ni foil were used for each exposure. The ( $hk0$ ) and ( $hkl$ ) zones were recorded from a prismatic crystal having nearly equidimensional cross section, approximately  $0.1 \times 0.1$  mm. A comparison strip of intensities was prepared by recording a given reflection from the crystal for varying known lengths of time, using the same experimental set-up and crystal as were used in preparing the Weissenberg patterns. The estimated intensities were converted to  $|F_{hkl}|^2$  values through the use of the Lorentz- and polarization-factor tables of Buerger & Klein (1945) for the  $hk0$ 's and the  $Lp$  chart of Cochran (1948) for the  $hkl$ 's. No attempt was made to correct for absorption effects, which were assumed to be relatively small owing to the small cross-sectional size of the crystal used and to use of  $\text{Mo } K\alpha$  radiation.

*Other considerations*

In the initial stages of the analysis the observed and calculated structure factors were related by use of the scaling constant  $k$ , where  $k\sum|F_o| = \sum|F_c|$ . Subsequently the relation  $k|F_o| = |F_c| \exp[-B(\sin^2 \theta)/\lambda^2]$  was used to fix the absolute scale of the observed structure factors and the value of the coefficient  $B$  of the temperature factor. For the final values of the coordinates,  $B = 1.22$  Å<sup>2</sup> for the ( $hk0$ ) zone and  $0.71$  Å<sup>2</sup> for the ( $hkl$ ) zone.

The Hartree atomic scattering curve for  $\text{O}^{-2}$  was used for the oxygen atoms and for the water molecule. For  $\text{K}^+$  a scattering curve corresponding to the

Thomas-Fermi values for K for  $(\sin \theta)/\lambda \geq 0.1 \text{ \AA}^{-1}$  and smoothed in to  $f = 18$  for  $(\sin \theta)/\lambda = 0$  was used. At the beginning of the structural analysis a curve prepared in an analogous fashion for  $V^{+5}$  was used. It was later found that significant improvement in the agreement between calculated and observed structure factors at small  $(\sin \theta)/\lambda$  values was obtained when the Thomas-Fermi values for V were used. The subsequent refinement was made using these values. All values of the scattering factors were taken from the *International Tables* (1935).

Maxima on the electron-density maps used in determining the structure were located by the method of Booth (1948).

### Determination and refinement of the structure

The structural problem consists in determining the parameters of one V, one K, three O, and one  $H_2O$  in the positions 4(c) of the space group  $Pnam$  (*International Tables*, 1935). The relatively short  $c$  axis suggested the use of the Patterson projection on (001) for the determination of the essential features of this structure, and accordingly this projection was prepared with the  $|F_{hko}|^2$  values on an arbitrary basis and the  $|F_{000}|^2$  term omitted. Buerger (1951) has shown how the Patterson projection on (001) for a crystal of similar symmetry and dimensions, berthierite,  $FeSb_2S_4$  ( $Pnam$ ,  $a = 11.44$ ,  $b = 14.12$ ,  $c = 3.76 \text{ \AA}$ ,  $Z = 4$ ), may be converted to an approximate electron-density map through the use of his minimum-function analysis. Buerger's procedure for  $FeSb_2S_4$  was followed for  $KVO_3 \cdot H_2O$  and the approximate  $\rho_z(x, y)$  map shown in Fig. 2 was obtained.\* From this map  $x$  and  $y$

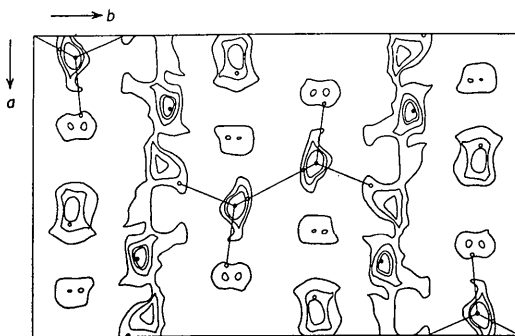


Fig. 2. Approximate  $\rho_z(x, y)$  map obtained by the minimum-function method. The final atomic positions are indicated by the small circles.

coordinates for the two heavy atoms K and V were assigned and structure factors  $F_{hko}$  calculated using the same atomic scattering curve for both atoms, that of  $K^+$ . These coordinates are given in column (1) of

\* With the  $|F_{000}|^2$  term omitted it was necessary to contour all the levels of the Patterson map, including the negative ones, in order to finish with a meaningful approximate  $\rho_z(x, y)$  map.

Table 2. Atomic positional parameters for  $KVO_3 \cdot H_2O$

Para- meters†	Stage of refinement*				
	(1)	(2)	(3)	(4) (final)	
V	$x$	0.07	0.073	0.074	0.074
	$y$	0.08	0.080	0.082	0.082
K	$x$	-0.23	-0.244	-0.244	-0.244
	$y$	-0.28	-0.279	-0.278	-0.278
O <sub>I</sub>	$x$	—	-0.003	-0.005	-0.005
	$y$	—	0.198	0.195	0.192
O <sub>II</sub>	$x$	—	0.267	0.276	0.277
	$y$	—	0.100	0.095	0.096
O <sub>III</sub>	$x$	—	-0.058	-0.046	-0.041
	$y$	—	-0.042	-0.048	-0.047
$H_2O$	$x$	—	0.125	0.119	0.118
	$y$	—	0.420	0.414	0.412
$R$ ‡		0.45	0.316	0.159	0.142 ( $hk0$ )
				0.194	§ ( $hk1$ )

\* See text for description of stage of refinement.

† All atoms in the asymmetric unit at  $z = \frac{1}{2}$ .

‡ Discrepancy factor.

§ Not calculated.

Table 2, which lists the coordinates and the discrepancy factor  $R$  found for each stage of the refinement. Signs calculated on the basis of these coordinates permitted the evaluation of  $\rho_z(x, y)$  using 87 terms. From this map, coordinates for all the atoms, but not for the water molecule, were assigned and a second  $\rho_z(x, y)$  containing 136 terms was calculated. Actually, a peak corresponding to the water molecule appeared on the first  $\rho_z(x, y)$  map as well as on the approximate  $\rho_z(x, y)$  map derived from the Patterson function. As it was believed that the compound was anhydrous, this peak was dismissed as being spurious and was expected to disappear in subsequent electron-density refinement, but at the completion of the second  $\rho_z(x, y)$  map, it was realized that the peak was real and that the compound was a monohydrate. This was fully verified, first, by the completed structural analysis and subsequently by chemical analysis, as explained previously. The coordinates assigned on the basis of the second  $\rho_z(x, y)$  map are given in Table 2, column (2). Two successive  $\rho_z(x, y)$  maps were then prepared, the second of which contained 152  $F_{hko}$ 's, on an absolute scale, corresponding to all of the non-zero intensities measured. Parameters derived from this last map were then used in fixing the signs of the  $F_{hkl}$ 's entering in a bounded electron-density projection described below.

All of the atoms are well resolved in the electron-density projection  $\rho_z(x, y)$  except V and O<sub>III</sub>. To obtain parameters for these atoms and to check the parameters of the other atoms, the projection on (001) of the electron density between  $z = 0$  and  $z = \frac{1}{2}$  was prepared, following the method of Booth (1948). The expression for the bounded projection of interest here has the form

$$S(x, y) = \frac{1}{2} \left[ \frac{1}{A} \sum_{h=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} F_{hko} \cos 2\pi(hx+ky) - \frac{2}{\pi A} \sum_{h=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{l=2n+1}^{\infty} \frac{F_{hkl}}{l} \sin 2\pi(hx+ky) \right]. \quad (1)$$

The first sum within the brackets of equation (1) is simply the usual electron-density projection on (001); hence the equation may be rewritten as

$$S(x, y) = \frac{1}{2} \left[ \rho_z(x, y) - \frac{2}{\pi A} S'(x, y) \right], \quad (2)$$

where

$$S'(x, y) = \sum_{h=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{l=2n+1}^{\infty} \frac{F_{hkl}}{l} \sin 2\pi(hx+ky). \quad (3)$$

For the space group  $Pnam$ , equation (3) reduces to

$$\frac{S'(x, y)}{4} = \sum_{h=0}^{\infty} \sum_{k=0}^{h+k=2n} F'_{hk} \sin 2\pi hx \cos 2\pi ky + \sum_{h=0}^{\infty} \sum_{k=0}^{h+k=2n+1} F'_{hk} \cos 2\pi hx \sin 2\pi ky, \quad (4)$$

where

$$F'_{hk} = \sum_{l=-\infty}^{\infty} \frac{F_{hkl}}{l}.$$

In evaluating  $S'(x, y)$  the  $F_{hkl}$  values for  $l \neq 1$  ( $l = 2n+1$ ) were derived from the  $F_{hkl}$  values in the following way: it was assumed that within a sufficient degree of approximation the shape of the scattering curves of the atoms involved is the same as that of some average reference atom. In the present case the reference scattering curve was taken as the average of the curves of  $K^+$  and  $V$  because these atoms contribute much more than the  $O$  atoms to the scattering. If one writes the atomic scattering factor in the form

$$f_i(hkl) = Z_i g(hkl),$$

where  $g(hkl)$  defines the shape of the reference scattering curve, then for  $Pnam$  and  $z = \frac{1}{2}$  for the atoms of the asymmetric unit, it follows that

$$\frac{F_{hkl}}{g(hkl)} = -\frac{F_{hk3}}{g(hk3)} = \frac{F_{hk5}}{g(hk5)} = \dots = (-1)^{\frac{n-1}{2}} \left[ \frac{F_{hkn}}{g(hkn)} \right].$$

The function  $S'(x, y)$  was evaluated with the magnitudes of the  $F_{hkl}$ 's on an arbitrary scale, the signs being calculated from the atomic parameters obtained from the last  $\rho_z(x, y)$  map. The scale of  $S'(x, y)$  was adjusted to make the average electron density of the bounded projection zero in regions where the heavy atoms do not appear in this projection. The parameters derived from this first bounded projection are given in column (3) of Table 2.

The bounded projection used here involves the difference of two separate series, the first a cosine series having as coefficients the  $F_{hko}$  values and the second a sine series with the  $(F_{hkl})/l$  values as coefficients. Distortion is introduced into the bounded projection if two series of unequal length are used, i.e. if the cosine and sine series are not terminated at the same value of  $(\sin \theta)/\lambda$ . A second source of distortion arises if the experimental threshold values of the observed  $F_{hkl}$  values are appreciably different for the  $(hk0)$  and  $(hkl)$  zones, since an imbalance in the number of terms of small magnitude in each of the two series results.

With these facts in mind, a second bounded projection was evaluated. The  $F_{hkl}$  values were put in on an absolute basis and no  $F_{hkl}$  term for which  $(\sin \theta)/\lambda > 0.7 \text{ \AA}^{-1}$  was used. For each  $F_{hkl}$  observed to be absent, for reflections up to and including  $(\sin \theta)/\lambda = 0.7 \text{ \AA}^{-1}$ , the experimentally determined threshold value was substituted. This second bounded projection (Fig. 3(a)) is relatively free from distortion and considerably improved in this respect over the first one. The  $\rho_z(x, y)$  map used in the preparation of this bounded projection is shown in Fig. 3(b).

Finally, a least-squares analysis of the  $x$  and  $y$  parameters of  $O_I$ ,  $O_{II}$ , and  $O_{III}$  was carried out using unweighted coefficients, and based on the parameters of column (3), Table 2. The final parameters are given

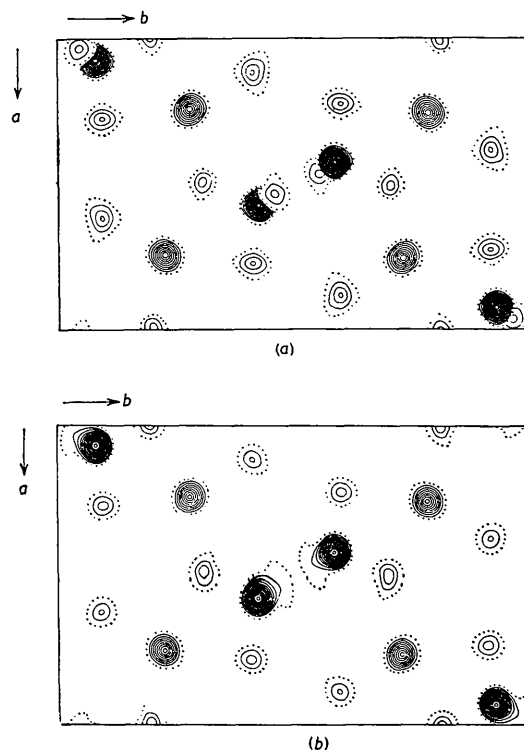


Fig. 3. (a) Projection on (001) of the superimposed electron-density maps taken between  $z = 0$  and  $z = \frac{1}{2}$  and between  $z = \frac{1}{2}$  and  $z = 1$  for  $KVO_3 \cdot H_2O$ . Contoured at intervals of  $4 \text{ e. \AA}^{-2}$ , with the dotted contour equal to  $4 \text{ e. \AA}^{-2}$ . (b) Electron-density projection  $\rho_z(x, y)$  for  $KVO_3 \cdot H_2O$ . Contoured in same way as (a).

in column (4), Table 2. For  $O_I$ ,  $O_{II}$ , and  $O_{III}$  these were obtained by applying the least-squares corrections. For the K, V, and  $H_2O$  parameters, the data of all of the electron-density projections were considered in arriving at the best choice. The standard errors associated with the oxygen atom parameters obtained from the least-squares analysis are very nearly the same for the three atoms; the averages are  $\epsilon_x = 0.016 \text{ \AA}$  and  $\epsilon_y = 0.018 \text{ \AA}$ . It was assumed that the limiting error in the V and K parameters was that of fixing the peak positions from the electron-density maps. Assuming this to be a maximum of 0.001 in cycles, the corresponding standard errors are  $\epsilon_x = 0.004 \text{ \AA}$  and  $\epsilon_y = 0.007 \text{ \AA}$ . The above errors lead to standard errors in the bond lengths of approximately  $\pm 0.02 \text{ \AA}$  for V-O and K-O bonds and  $\pm 0.03 \text{ \AA}$  for O-O bonds. The corresponding error in O-V-O bond angles is about  $\pm 1^\circ$ . The precise positioning of the  $H_2O$  molecule was not considered to be of any real importance; the standard errors of its  $x$  and  $y$  parameters are of the order of  $0.03 \text{ \AA}$ .

Throughout the course of the structure analysis, plots of  $R'$  versus  $\sin \theta$ , as suggested by Luzzati (1952), were found to be very helpful in deciding whether the structure was converging. Such a plot, based on the final parameters and compared with the corresponding theoretical curves of Luzzati, indicates a maximum mean error in bond length,  $|\Delta r| = 0.04 \text{ \AA}$ , entirely in agreement with the least-squares results.

### Description and discussion of the structure

A pictorial view of the structure of  $KVO_3 \cdot H_2O$  is given in Fig. 5, and a projected view in Fig. 4(a). It is seen

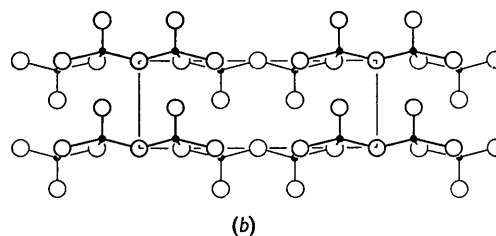
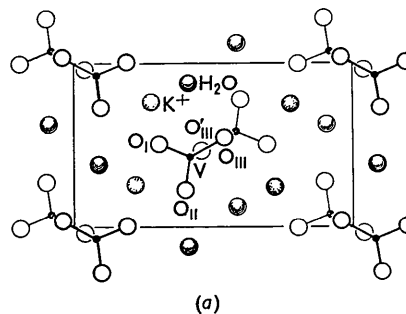


Fig. 4. (a) Structure of  $KVO_3 \cdot H_2O$  projected on (001).  
(b) Structure of  $V_2O_5$  projected on (001) (after Byström *et al.*, 1950).

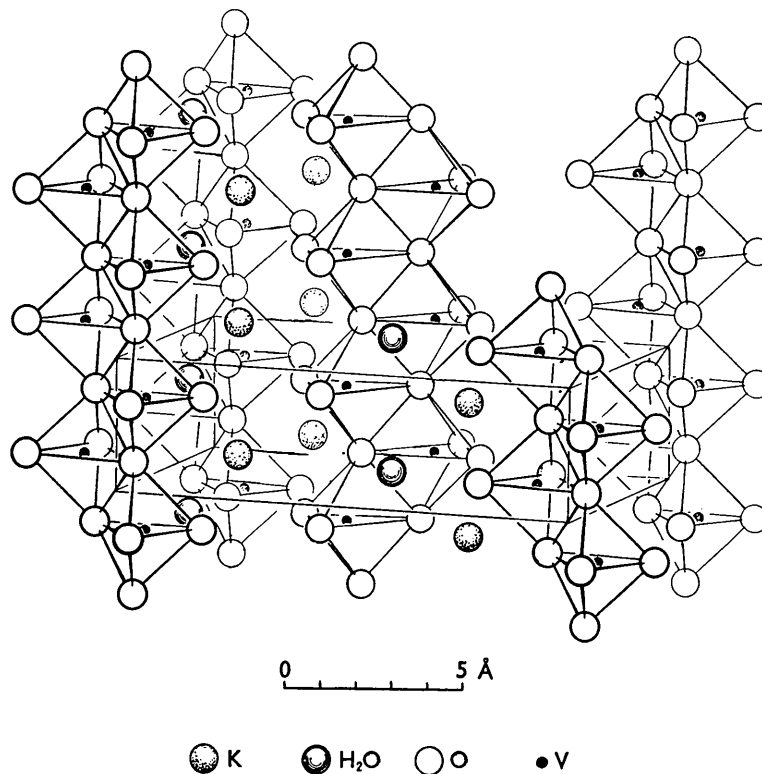


Fig. 5. Pictorial view of  $KVO_3 \cdot H_2O$ .

that each vanadium atom is linked to five oxygen atoms to form a distorted trigonal dipyramid. The trigonal dipyramidal polyhedra share edges to form continuous chains parallel to the *c* axis, accounting for the observed pronounced fibrous cleavage. The coordination of oxygen atoms around the vanadium atoms is shown in detail in Fig. 6; the corresponding

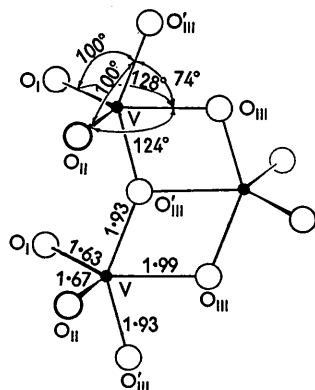


Fig. 6. Details of vanadium-oxygen coordination for  $\text{KVO}_3 \cdot \text{H}_2\text{O}$ .

V-O bond lengths and angles and other important bond lengths and angles in the structure are listed in Table 3. As shown in the projected view of Fig. 4(a),

Table 3. Bond lengths and bond angles for  $\text{KVO}_3 \cdot \text{H}_2\text{O}$

Bond lengths (Å)		Bond angles (°)	
V-O <sub>I</sub>	1.63	O <sub>I</sub> -V-O <sub>II</sub>	106
V-O <sub>II</sub>	1.67	O <sub>I</sub> -V-O <sub>III</sub>	128
V-O <sub>III</sub>	1.93 (2)	O <sub>II</sub> -V-O <sub>III</sub>	124
V-O <sub>III</sub>	1.99	O <sub>II</sub> -V-O <sub>I</sub>	147
(V-O bonds $\pm 0.02$ Å)		O <sub>I</sub> -V-O <sub>III</sub>	100
		O <sub>II</sub> -V-O <sub>III</sub>	100
		(all $\pm 1^\circ$ )	
O <sub>I</sub> -O <sub>II</sub>	2.64		
O <sub>I</sub> -O <sub>III</sub>	2.73		
O <sub>II</sub> -O <sub>III</sub>	2.75		
O <sub>III</sub> -O <sub>III</sub>	2.34 (2)		
(O-O bonds $\pm 0.03$ Å)			
K-O <sub>I</sub>	2.98 (2), 2.79 (2)		
K-O <sub>II</sub>	3.10 (2)		
K-H <sub>2</sub> O	2.79 (2)		
(K-O bonds $\pm 0.02$ Å; K-H <sub>2</sub> O bonds $\pm 0.04$ Å)			
V-V	3.14 $\pm 0.02$		

O<sub>I</sub>, O<sub>II</sub>, and O<sub>III</sub> lie at the vertices of a triangle containing the vanadium atom, which is displaced away from the center of the triangle toward the edge O<sub>I</sub>O<sub>II</sub>. The V-O bonds lying in the triangle are: V-O<sub>I</sub> = 1.63, V-O<sub>II</sub> = 1.67, V-O<sub>III</sub> = 1.99 Å. The plane defined as that plane containing the vanadium atoms and parallel to the *c* axis also contains the O<sub>III</sub> atoms. If the bond lengths V-O<sub>I</sub> and V-O<sub>II</sub> and the bond angles O<sub>I</sub>-V-O<sub>III</sub> and O<sub>II</sub>-V-O<sub>III</sub> were the same, respectively, this plane would then be a plane of symmetry. The lengths V-O<sub>I</sub> and V-O<sub>II</sub> were found to be  $1.63 \pm 0.02$  and  $1.67 \pm 0.02$  Å, respectively. A rough statistical calculation shows that there is high prob-

ability that the V-O<sub>II</sub> bond is truly longer than the V-O<sub>I</sub> bond. This conclusion is supported by the difference in bond angles found. It seems probable, therefore, that the chain does not conform exactly to a plane of symmetry. Detailed examination of the structure shows that the O<sub>I</sub>-K and O<sub>II</sub>-K lengths are different: K-O<sub>I</sub> is  $2.79 \pm 0.02$  Å, and K-O<sub>II</sub> is  $3.10 \pm 0.02$  Å. Thus it seems likely that the small departures from planar symmetry in the chain are due to packing effects, which result in overall lowering of the lattice energy. The O<sub>III</sub>-O<sub>III</sub> distance of 2.34 Å is rather short and results from strong polarization of these atoms by the vanadium atoms. This postulated polarization is in agreement with the experimental observation that the atomic scattering curve for neutral vanadium gave better results than that for V<sup>+5</sup>.

Fivefold coordination in crystals is rare. The only analogous situation seems to be that of V<sub>2</sub>O<sub>5</sub> (Byström, Wilhelmi & Brotzen, 1950). In this compound there is a definitely similar distorted trigonal dipyramidal polyhedral chain linkage, as may be seen by comparing Fig. 4(a) and (b). The chains as found for  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  are further linked in V<sub>2</sub>O<sub>5</sub> through oxygen atoms to form the more condensed system. The bond lengths of interest in the two compounds are compared in Table 4. For what has here been designated the V-O<sub>I</sub>

Table 4. Comparison of bond lengths for  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  and V<sub>2</sub>O<sub>5</sub>

	$\text{KVO}_3 \cdot \text{H}_2\text{O}$	V <sub>2</sub> O <sub>5</sub>
V-O <sub>I</sub>	1.63 Å	$1.77 \pm 0.03$ Å
V-O <sub>II</sub>	1.67	$1.54 \pm 0.06$
V-O <sub>III</sub>	1.93	$1.88 \pm 0.04$
V-O <sub>III</sub>	1.99	$2.02 \pm 0.08$
O <sub>I</sub> -O <sub>II</sub>	2.64	2.63
O <sub>I</sub> -O <sub>III</sub>	2.73	2.70
O <sub>II</sub> -O <sub>III</sub>	2.75	2.73
O <sub>III</sub> -O <sub>III</sub>	2.34	2.39

bond, the length in V<sub>2</sub>O<sub>5</sub> (1.77 Å) is significantly longer than that in  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  (1.63 Å). This difference is to be expected because in V<sub>2</sub>O<sub>5</sub> it is the O<sub>I</sub> atom that links the chains together to form sheets. The bond in V<sub>2</sub>O<sub>5</sub> corresponding to the V-O<sub>II</sub> bond of  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  is short, being only 1.54 Å in length, and must therefore be highly polarized. The differences in the configurations of the chains of the two compounds are such that, despite the much smaller V-O bond length in V<sub>2</sub>O<sub>5</sub>, the smallest O-O separations are nearly the same for the two compounds. The other bonds common to the two compounds have very nearly the same lengths. It is interesting that V<sub>2</sub>O<sub>5</sub> is yellow red, whereas  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  is colorless. It is difficult to decide what differences in the bonding of the two compounds lead to this difference of absorption in the visible spectrum. It is hoped that the results of crystal-structure analysis of other vanadates being made in this and other laboratories will permit this question to be resolved later.

The K<sup>+</sup> of  $\text{KVO}_3 \cdot \text{H}_2\text{O}$  is surrounded by six oxygen

Table 5. Observed and calculated structure factors  $hk0$  and  $hk1$   
 Values of  $F_c$  for  $hk0$  are based on the atomic coordinates of column (4),  
 Table 2; those for  $hk1$  on the coordinates of column (3), Table 2.

$hk0$	$F_o$	$F_c$	$hk0$	$F_o$	$F_c$	$hk0$	$F_o$	$F_c$	$hk0$	$F_o$	$F_c$	$hk0$	$F_o$	$F_c$	$hk0$	$F_o$	$F_c$
000		304	210	48.4	-45.4	3,20,0		3.6	5,19,0	9.2	5.2	610	11.5	12.8	10,9,0	15.6	-18.5
200	15.4	13.5	220	69.6	69.0	3,21,0		5.8	5,20,0	11.5	13.2	620	29.2	-38.5	10,10,0		-4.2
400	55.8	51.3	230	75.0	-78.7	3,22,0	17.4	16.4	5,21,0		4.6	630	17.7	1.0	10,11,0		-7.9
600	63.7	-69.1	240	99.2	-93.5	4,0,0		17.2	6,0,0		1.4	640	20.2	23.3	10,12,0		7.3
800	6.1	7.8	250	47.4	-43.9	410		33.3	410		-29.8	650	11.5	10.0	10,13,0		9.6
10,0,0	7.2	-14.1	260	20.5	8.1	420		56.3	620		0.1	660	11.3	10.3	10,14,0		-7.1
12,0,0	16.6	17.9	270	46.6	-43.2	430		72.2	630		1.4	670	11.5	10.0	10,15,0		6.0
			280	46.6	-43.2	440		36.6	640		0.1	680	11.5	10.0			
020	10.2	9.8	290	43.3	36.5	450		39.7	650	12.3	-16.2	690	15.9	-13.9	11,1,0	11.8	9.0
040	16.6	-32.0	300	21.0	19.2	460		16.7	660	12.3	-16.2	700	15.9	-13.9	11,2,0	14.3	14.3
060	105.2	-111.6	310	22.5	22.5	470		22.5	670	7.2	-8.5	710	11.0	-5.4	11,3,0		-8.7
080	55.0	-46.7	320	22.3	21.6	480		2.2	680	7.7	6.3	720	17.7	-18.6	11,4,0	20.7	22.0
0,10,0	39.9	39.7	330	9.0	-9.5	490		36.2	690	11.5	-14.5	730	11.0	-5.1	11,5,0		8.2
0,12,0	28.9	22.3	340	25.0	-25.0	500	35.8	35.8	700	11.5	-14.5	740	11.0	-5.1	11,6,0		-9.5
0,14,0	25.9	26.2	350	25.3	-22.8	4,11,0	25.1	18.9	6,11,0		5.4	750	14.3	13.0	11,7,0	14.1	-12.6
0,16,0	20.7	-29.8	360	10.8	19.0	4,12,0	24.1	-27.7	6,12,0	22.8	-17.9	760	11.0	-5.6	11,8,0		-3.8
0,18,0	17.7	8.2	370	21.0	-8.2	4,13,0	14.6	-13.6	6,13,0		9.7	770	11.0	6.3	11,9,0		8.4
0,20,0	8.4	-12.7	380	21.8	20.5	4,14,0		4.2	6,14,0	20.7	-23.9	780	14.6	12.7	11,10,0	19.7	-23.3
0,22,0		18.1	390	24.9	-24.9	4,15,0	20.5	-17.3	6,15,0		-1.5	790	18.7	-22.9	11,12,0		7.7
			400	22.0	14.1	4,16,0		9.0	6,16,0		1.0	800	20.7	26.1	11,13,0		6.7
110	56.6	82.5	2,21,0		-3.4	4,17,0	9.0	-7.7	6,17,0		0.6	810	20.7	26.1	12,1,0		8.5
120	21.0	-22.3	2,22,0			4,18,0	9.0	17.3	6,18,0		0.6	820	20.7	26.1	12,2,0		-10.4
130	3.6	-2.2	3,0,0	56.8	59.6	4,19,0		10.2	6,19,0	16.1	15.0	830	20.7	26.1	12,3,0		0.4
140	73.0	-73.8	310	22.8	-14.9	4,20,0	15.6	10.4	7,0,0	25.6	-25.5	840	18.2	18.9	12,4,0		4.3
150	13.1	-8.0	320	35.0	-11.6	510	23.8	-20.1	720	6.7	-12.4	850	24.1	-34.1	12,5,0	8.4	6.7
160	41.2	41.7	330	42.8	-45.6	520	41.0	-36.4	730	12.5	-15.5	860	24.1	-34.1	12,6,0		-6.4
170	53.2	-46.9	340	15.9	-11.6	530	5.1	-4.7	740	6.8	8.4	870	24.1	-34.1	12,7,0		-1.5
180	15.6	-18.2	350	53.0	-50.7	540	65.5	-73.4	750	33.0	35.6	880	24.1	-34.1	12,8,0		-3.9
190	22.8	-19.5	360	37.0	-37.0	550	30.0	29.6	760	19.2	-25.7	890	24.1	-34.1	12,9,0		-5.9
1,10,0	40.2	45.6	370	35.6	-31.3	560	60.5	-60.5	770	15.7	-17.7	900	24.1	-34.1	13,0,0		7.2
1,11,0	22.5	18.8	380	44.3	45.2	570	21.5	20.1	780	14.6	16.8	910	11.0	8.2	13,1,0		-2.7
1,12,0	21.7	21.8	390	21.8	21.8	580		-2.1	790	14.6	16.8	920	11.0	8.2	13,2,0		-2.7
1,13,0	16.4	10.5	400	31.0	6.1	590		-2.1	800	14.6	16.8	930	11.0	8.2	13,3,0	9.5	8.7
1,14,0	13.3	3.7	410	31.0	6.1	600		-2.1	810	14.6	16.8	940	11.0	8.2	13,4,0		7.5
1,15,0	14.1	-11.1	420	31.0	6.1	610		-2.1	820	14.6	16.8	950	11.0	8.2	13,5,0		-1.0
1,16,0	14.3	-11.1	430	31.0	6.1	620		-2.1	830	14.6	16.8	960	11.0	8.2	13,6,0		-1.0
1,17,0	16.1	-12.0	440	31.0	6.1	630		-2.1	840	14.6	16.8	970	11.0	8.2	13,7,0		-1.2
1,18,0		8.9	450	31.0	6.1	640		-2.1	850	14.6	16.8	980	11.0	8.2	13,8,0		-1.2
1,19,0		8.9	460	31.0	6.1	650		-2.1	860	14.6	16.8	990	11.0	8.2	13,9,0		-1.2
1,20,0		-1.5	470	31.0	6.1	660		-2.1	870	14.6	16.8	1000	11.0	8.2	14,0,0		-1.2
1,21,0			480	31.0	6.1	670		-2.1	880	14.6	16.8	1010	11.0	8.2	14,1,0		-1.2
1,22,0			490	31.0	6.1	680		-2.1	890	14.6	16.8	1020	11.0	8.2	14,2,0		-1.2
1,23,0			500	31.0	6.1	690		-2.1	900	14.6	16.8	1030	11.0	8.2	14,3,0		-1.2
1,24,0			510	31.0	6.1	700		-2.1	910	14.6	16.8	1040	11.0	8.2	14,4,0		-1.2
1,25,0			520	31.0	6.1	710		-2.1	920	14.6	16.8	1050	11.0	8.2	14,5,0		-1.2
1,26,0			530	31.0	6.1	720		-2.1	930	14.6	16.8	1060	11.0	8.2	14,6,0		-1.2
1,27,0			540	31.0	6.1	730		-2.1	940	14.6	16.8	1070	11.0	8.2	14,7,0		-1.2
1,28,0			550	31.0	6.1	740		-2.1	950	14.6	16.8	1080	11.0	8.2	14,8,0		-1.2
1,29,0			560	31.0	6.1	750		-2.1	960	14.6	16.8	1090	11.0	8.2	14,9,0		-1.2
1,30,0			570	31.0	6.1	760		-2.1	970	14.6	16.8	1100	11.0	8.2	15,0,0		-1.2
1,31,0			580	31.0	6.1	770		-2.1	980	14.6	16.8	1110	11.0	8.2	15,1,0		-1.2
1,32,0			590	31.0	6.1	780		-2.1	990	14.6	16.8	1120	11.0	8.2	15,2,0		-1.2
1,33,0			600	31.0	6.1	790		-2.1	1000	14.6	16.8	1130	11.0	8.2	15,3,0		-1.2
1,34,0			610	31.0	6.1	800		-2.1	1010	14.6	16.8	1140	11.0	8.2	15,4,0		-1.2
1,35,0			620	31.0	6.1	810		-2.1	1020	14.6	16.8	1150	11.0	8.2	15,5,0		-1.2
1,36,0			630	31.0	6.1	820		-2.1	1030	14.6	16.8	1160	11.0	8.2	15,6,0		-1.2
1,37,0			640	31.0	6.1	830		-2.1	1040	14.6	16.8	1170	11.0	8.2	15,7,0		-1.2
1,38,0			650	31.0	6.1	840		-2.1	1050	14.6	16.8	1180	11.0	8.2	15,8,0		-1.2
1,39,0			660	31.0	6.1	850		-2.1	1060	14.6	16.8	1190	11.0	8.2	15,9,0		-1.2
1,40,0			670	31.0	6.1	860		-2.1	1070	14.6	16.8	1200	11.0	8.2	16,0,0		-1.2
1,41,0			680	31.0	6.1	870		-2.1	1080	14.6	16.8	1210	11.0	8.2	16,1,0		-1.2
1,42,0			690	31.0	6.1	880		-2.1	1090	14.6	16.8	1220	11.0	8.2	16,2,0		-1.2
1,43,0			700	31.0	6.1	890		-2.1	1100	14.6	16.8	1230	11.0	8.2	16,3,0		-1.2
1,44,0			710	31.0	6.1	900		-2.1	1110	14.6	16.8	1240	11.0	8.2	16,4,0		-1.2
1,45,0			720	31.0	6.1	910		-2.1	1120	14.6	16.8	1250	11.0	8.2	16,5,0		-1.2
1,46,0			730	31.0	6.1	920		-2.1	1130	14.6	16.8	1260	11.0	8.2	16,6,0		-1.2
1,47,0			740	31.0	6.1	930		-2.1	1140	14.6	16.8	1270	11.0	8.2	16,7,0		-1.2
1,48,0			750	31.0	6.1	940		-2.1	1150	14.6	16.8	1280	11.0	8.2	16,8,0		-1.2
1,49,0			760	31.0	6.1	950		-2.1	1160	14.6	16.8	1290	11.0	8.2	16,9,0		-1.2
1,50,0			770	31.0	6.1	960		-2.1	1170	14.6	16.8	1300	11.0	8.2	17,0,0		-1.2
1,51,0			780	31.0	6.1	970		-2.1	1180	14.6	16.8	1310	11.0	8.2	17,1,0		-1.2
1,52,0			790	31.0	6.1	980		-2.1	1190	14.6	16.8	1320	11.0	8.2	17,2,0		-1.2
1,53,0			800	31.0	6.1	990		-2.1	1200	14.6	16.8	1330	11.0	8.2	17,3,0		-1.2
1,54,0			810	31.0	6.1	1000		-2.1	1210	14.6	16.8	1340	11.0	8.2	17,4,0		-1.2
1,55,0			820	31.0	6.1	1010		-2.1	1220	14.6	16.8	1350	11.0	8.2	17,5,0		-1.2
1,56,0			830	31.0	6.1	1020		-2.1	1230	14.6	16.8	1360	11.0	8.2	17,6,0		-1.2
1,57,0			840	31.0	6.1	1030		-2.1	1240	14.6	16.8	1370	11.0	8.2	17,7,0		-1.2
1,58,0			850	31.0	6.1	1040		-2.1	1250	14.6	16.8	1380	11.0	8.2	17,8,0		-1.2
1,59,0			860	31.0	6.1	1050		-2.1	1260	14.6	16.8	1390	11.0	8.2	17,9,0		-1.2
1,60,0			870	31.0	6.1	1060		-2.1	1								